



Class 11 Maths || Chapter 5 : Complex Numbers and Quadratic Equations

|| Class Notes

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Class-11- Maths

Chapter 5

Complex Numbers

Theory →

$$\text{iota} = i = \sqrt{-1}$$

Not real

$$x^2 = -1$$

$$x = \pm \sqrt{-1} \Rightarrow x = \pm i$$

$i^1 = i$	$i^5 = i$	$i^9 = i$	$i^{4n+1} = i$ ★
$i^2 = -1$	$i^6 = -1$	$i^{10} = -1$	$i^{4n+2} = -1$ ★
$i^3 = -i$	$i^7 = -i$	$i^{11} = -i$	$i^{4n+3} = -i$ ★
$i^4 = 1$	$i^8 = 1$	$i^{12} = 1$	$i^{4n} = 1$ ★

$\pm 1 \quad \pm i$

$$i^{63} = i^{4 \times 15 + 3} = -i$$

Special Case : →

$$\boxed{\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}}$$

Condition:

at least one
of a & b
must be
non-negative.

$$\frac{+}{0}$$

$$a = -1 \quad \checkmark$$

$$b = -1 \quad \checkmark$$

$$\sqrt{-1} \cdot \sqrt{-1} = \sqrt{(-1) \cdot (-1)}$$

$$i \cdot i = \sqrt{+1}$$

$$i^2 = 1$$

$$\boxed{-1 = 1}$$

False

Ⓐ Ⓑ

$$++ \quad \sqrt{a} \sqrt{b} = \sqrt{ab} \quad \checkmark$$

$$+- \quad \sqrt{a} \sqrt{b} = \sqrt{ab} \quad \checkmark$$

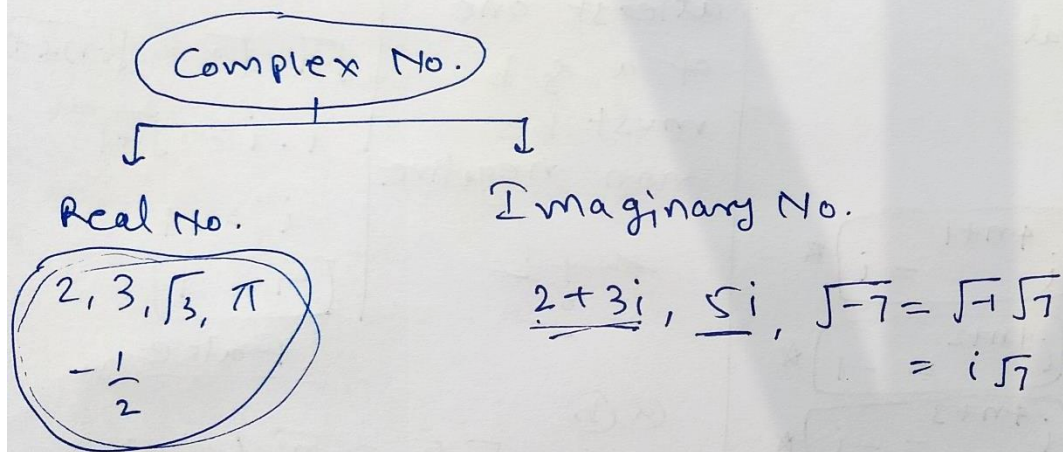
$$-+ \quad \sqrt{a} \sqrt{b} = \sqrt{ab} \quad \checkmark$$

$$\ominus \ominus \quad \sqrt{a} \sqrt{b} \neq \sqrt{ab} \quad \times$$



Complex No.

$2 \rightarrow \text{Real No.}$
 $5i \rightarrow \text{imaginary}$
 $2+3i \rightarrow \text{imaginary No.}$



Complex No. $z = x+iy$ $\begin{matrix} x \in \mathbb{R} \\ y \in \mathbb{R} \end{matrix}$

Real part of z
 $= \text{Re}(z)$
 $= x$

Imaginary part of z
 $= \text{Im}(z)$
 $= y$

$2 \rightarrow \text{Rat?}$

$5\sqrt{3} \rightarrow \text{Irr.}$

$2+5\sqrt{3} \rightarrow \text{Irr.}$

Not #

Complex No.

$$z = x+iy$$

Purely Real No.

$$z = x, y=0$$

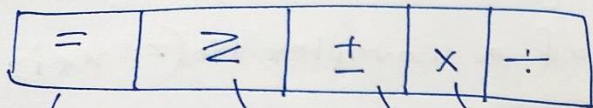
$x \in \mathbb{R}$

Purely Imaginary No.

$$z = iy$$

$x=0$

Algebra of Complex Numbers.



$$i^2 = -1$$

$$\frac{2+\sqrt{3}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}}$$

Equality

$$z_1 = z_2$$

$$x_1 + iy_1 = x_2 + iy_2$$

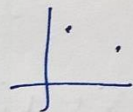
$$x_1 = x_2$$

$$y_1 = y_2$$

Inequality
in complex
numbers
does not
make
any
SENSE

$$2+3i \nless 100+i1000$$

Point Point



$$z_1 \pm z_2 = (x_1 + iy_1) \pm (x_2 + iy_2) = (x_1 \pm x_2) + i(y_1 \pm y_2)$$

$$z_1 \cdot z_2 = (x_1 + iy_1) \cdot (x_2 + iy_2) = x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$

$$i^2 = -1$$

Note: Multiplicative inverse of

$$z = \frac{1}{z}$$

e.g.

$$M I \text{ of } 2+3i = \frac{1}{2+3i}$$

eg. Simplify $\frac{1}{2+i3}$ into $a+ib$.

$$\frac{1}{2+i3} = \frac{1}{2+i3} \times \frac{2-i3}{2-i3}$$

$$(a+b) \cdot (a-b) = a^2 - b^2$$

$$= \frac{2-3i}{2^2 - (i3)^2}$$

$$= \frac{2-3i}{4 - (-1)9}$$

$$= \frac{2-3i}{13}$$

$$= \frac{2}{13} - \frac{3i}{13} = a+ib$$

$$a = \frac{2}{13}, b = -\frac{3}{13}$$

Tools, Modulus + Argument + Conjugate

Modulus of a complex No ($z = x+iy$)
 $= |z| = |x+iy| = \sqrt{x^2 + y^2}$

$$|z| = \sqrt{[\text{Re}(z)]^2 + [\text{Im}(z)]^2}$$

Conjugate of a complex No. ($z = x+iy$)

$$\bar{z} = \overline{x+iy} = x-yi$$

e.g. $z = 5-4i$

$$|z| = \sqrt{5^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$

$$\bar{z} = \overline{5-4i} = 5+4i$$

Properties of modulus & Conjugate

$$\textcircled{1} \quad \boxed{z \cdot \bar{z} = |z|^2} \quad \star$$

$$\textcircled{2} \quad \star \quad \boxed{|z_1 \cdot z_2| = |z_1| \cdot |z_2|}, \quad \overline{(z_1 \cdot z_2)} = \bar{z}_1 \cdot \bar{z}_2$$

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}, \quad \overline{\left(\frac{z_1}{z_2} \right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

$\times$

$$\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$$

$\times$

$$\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$$

Proof:

$$\overbrace{z} \cdot \underbrace{\bar{z}} = |z|^2$$

$$\Rightarrow (x+iy) \cdot (x-iy) = \left(\sqrt{x^2+y^2} \right)^2$$

$$\Rightarrow (x)^2 - (iy)^2 = x^2 + y^2$$

$$\Rightarrow \underline{\underline{x^2 + y^2 = x^2 + y^2}}$$

Note:

Mult. Inverse of z

$$= \frac{1}{z}$$

$$= \frac{1}{z} \times \frac{\bar{z}}{\bar{z}}$$

$$= \frac{\bar{z}}{|z|^2}$$

$\star$

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Class-11-Maths Complex Numbers Chapter-5

Exercise 5.1

i = imaginary

$$\left. \begin{array}{l} i = \sqrt{-1} \\ i^2 = -1 \\ i^3 = -i \\ i^4 = 1 \end{array} \right\} \left. \begin{array}{l} i^{4n+1} = i \\ i^{4n+2} = -1 \\ i^{4n+3} = -i \\ i^{4n} = 1 \end{array} \right\} n \in \mathbb{I}$$

Q.1. $(\cancel{8}i) \cdot \left(-\frac{3}{\cancel{8}}i \right) = -3 \cdot (i^2) = -3 \cdot (-1) = 3 = \underline{3+io}$

Q.2. $i + i^{19} = i - i = 0 = 0+io$

$i^{19} = i^{16+3} = i^{4 \times 4 + 3} = -i$

(a+ib)

Q.3. $i^{-39} = \frac{1}{i^{39}} \times \frac{i}{i} = \frac{i}{i^{40}} = \frac{i}{1} = i = \underline{0+1i}$

Q.4

$$\begin{aligned} & 3(7+i7) + i(7+i7) \\ &= (21) + i(21 + 7i) + (i^2 \cdot 7) \\ &= \underline{21} + 28i + (-1) \cdot 7 \\ &= 14 + 28i = a+ib \end{aligned}$$

Q.5. $(1-i) - (-1+i6)$

$$\begin{aligned} &= 1-i+1-i \cdot 6 \\ &= 2-i \cdot 7 \\ &= 2-7i \end{aligned}$$

Q.6. $\left(\frac{1}{5} + i \frac{2}{5} \right) - \left(4 + i \frac{5}{2} \right)$

$$\begin{aligned} &= \frac{1}{5} - 4 + i \left(\frac{2}{5} - \frac{5}{2} \right) \\ &= \left(\frac{-19}{5} \right) + i \left(\frac{-21}{10} \right) \end{aligned}$$

$$\textcircled{7} \left[\left(\frac{1}{3} + i\frac{7}{3} \right) + \left(4 + i\frac{1}{3} \right) \right] - \left(-\frac{4}{3} + i \right)$$

$$= \underbrace{\left(\frac{1}{3} + 4 + \frac{4}{3} \right)}_{\text{Real part}} + i \underbrace{\left(\frac{7}{3} + \frac{1}{3} - 1 \right)}_{\text{Im. part}}$$

$$= \frac{1+12+4}{3} + i \left(\frac{7+1-3}{3} \right)$$

$$= \frac{17}{3} + i \left(\frac{5}{3} \right) = a+ib$$

$a = \frac{17}{3}, b = \frac{5}{3}$

$$\textcircled{8} (1-i)^4$$

$$= (1-i)^2 \cdot (1-i)^2$$

$$= (1^2 + i^2 - 2 \cdot 1 \cdot i) \cdot (1-i)^2 \quad (a-b)^2 = a^2 + b^2 - 2ab$$

$$= (\cancel{1} - \cancel{1} - 2i) \cdot (1-i)^2 = (-2i) \cdot (-2i) = (+4)(i^2) = -4$$

$$(a-b)^4 = (a-b)^2 \cdot (a-b)^2$$

$$= a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\textcircled{9} \left(\frac{1}{3} + 3i \right)^3$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\left(\frac{1}{3} + 3i \right)^3 = \left(\frac{1}{3} \right)^3 + 3 \left(\frac{1}{3} \right)^2 (3i) + 3 \left(\frac{1}{3} \right) (3i)^2 + (3i)^3$$

$$= \frac{1}{27} + i - 9 - 27i$$

$$= \frac{1-243}{27} - 26i = -\frac{242}{27} - 26i$$

$$\textcircled{10} \left(-2 - \frac{1}{3}i \right)^3 = - \left(2 + \frac{1}{3}i \right)^3$$

$$= - \left[2^3 + 3 \cdot 2^2 \cdot \frac{i}{3} + 3 \cdot 2 \cdot \left(\frac{i}{3} \right)^2 + \left(\frac{i}{3} \right)^3 \right]$$

$$= - \left[8 + 4i - \frac{2}{3} - \left(\frac{i}{27} \right) \right]$$

$$= - \left(\frac{22}{3} + \frac{108-i}{27} i \right)$$

$$= -\frac{22}{3} - \frac{107}{27} i$$

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Class-11-Maths Complex Number

Exercise 5.1

Q.11 multiplicative inverse of $4-3i$ = $\frac{1}{4-3i}$

multiplicative inverse of $z = \frac{1}{z}$

$z = x + iy$

$$i^2 = -1$$

$$(2+\sqrt{3}) \rightarrow (2-\sqrt{3})$$

$$= \frac{1}{4-3i} \times \frac{4+3i}{4+3i}$$

$i \times i = -1$ Real

$$= \frac{4+3i}{(4)^2 - (3i)^2} = \frac{4+3i}{16 - (9(-1))}$$

$$= \frac{4+3i}{16+9} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3i}{25}$$

(12)

multiplicative Inverse of $\sqrt{5}+3i$

$$= \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$$

$$= \frac{\sqrt{5}-3i}{(\sqrt{5})^2 - (3i)^2}$$

$$= \frac{\sqrt{5}-3i}{5 - 9(i^2)}$$

$$= \frac{\sqrt{5}-3i}{5 - 9(-1)} = \frac{\sqrt{5}-3i}{5+9}$$

$$= \frac{\sqrt{5}-3i}{14}$$

(13) multiplicative inverse
of $(-i) = \frac{1}{-i} \times \frac{i}{i}$

$$= \frac{i}{-\cancel{i^2}} = \frac{i}{-(-1)}$$

$$= \frac{i}{+1} = i$$

(14) $a+ib = \frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i) - (\sqrt{3}-i\sqrt{2})}$

$(x+y)(x-y)$
 $= x^2 - y^2$

$$= \frac{(3)^2 - (i\sqrt{5})^2}{\cancel{\sqrt{3}} + \sqrt{2}i - \cancel{\sqrt{3}} + i\sqrt{2}}$$

$$= \frac{9 - \cancel{i^2} \cdot 5}{2\sqrt{2}i}$$

$$= \frac{(9+5)}{2\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i}$$

$$= \frac{14\sqrt{2}i}{2 \cdot 2 \cdot (-1)}$$

$$= \frac{-7\sqrt{2}i}{2} = 0 - \frac{7\sqrt{2}}{2}i$$

Argand plane

Modulus ✓

Conjugate ✓

Argument + EXTRA

Polar form

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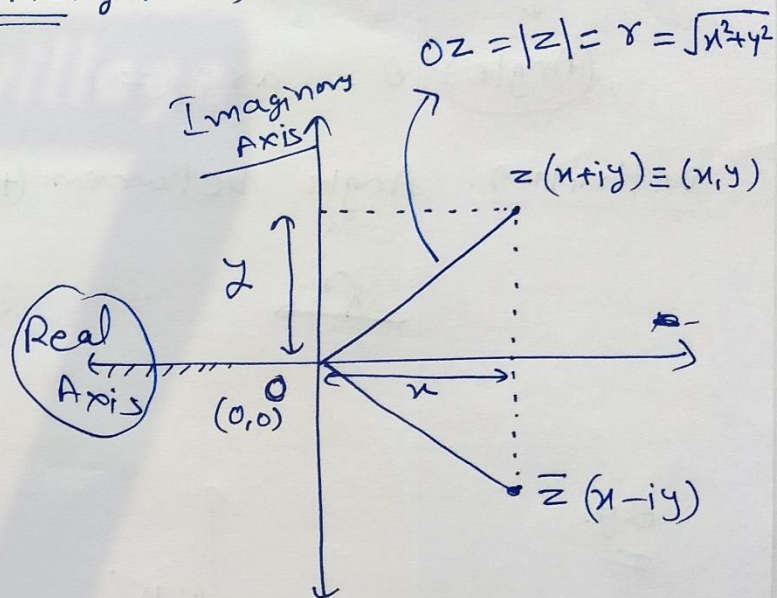
Complex Number (Geometry Part)

$z = x + iy$ (x, y)
Number point

$|z| = \sqrt{x^2 + y^2}$ = distance between $z(x + iy)$ and $0(0 + i0)$

$\bar{z} = x - iy$ $(x, -y)$

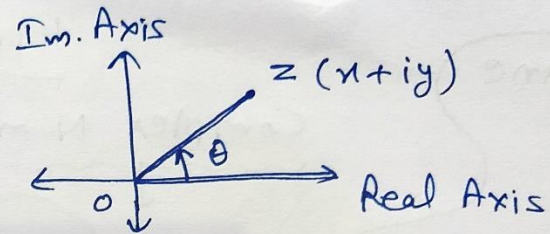
$\bar{z}$ = mirror image of 'z' in the mirror "Real Axis".



Argand plane
(Complex plane)
(Gaussian plane)

Argument of a Complex Number = θ

Angle $\neq \theta = \arg(z)$



Definition: Angle between + Real Axis

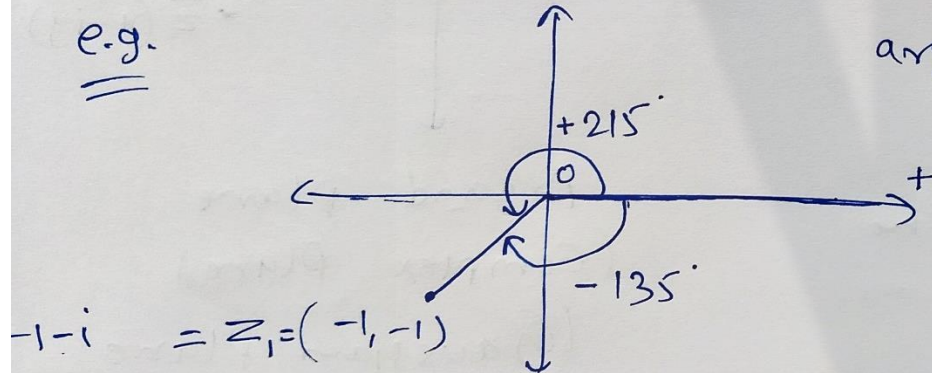


Initial Ray

& line segment joining $O(0+io)$ and $Z(x+iy)$

Final Ray

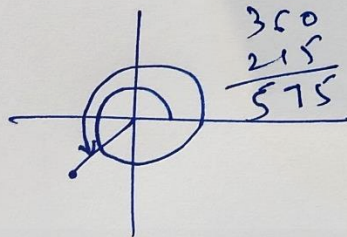
e.g.



$$\arg(z_1) = \arg(-1 - i)$$

$$= 215^\circ, -135^\circ, +575^\circ$$

P.A

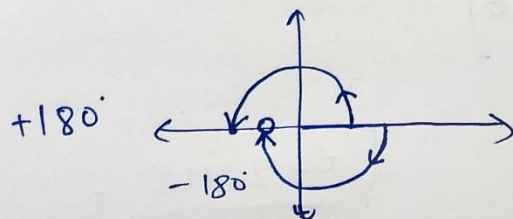


Concept of Principal Argument (θ)

Amplitude

$$-\pi < \theta \leq \pi$$

$$-180^\circ < \theta \leq +180^\circ$$



Steps to find Principal Argument θ :-

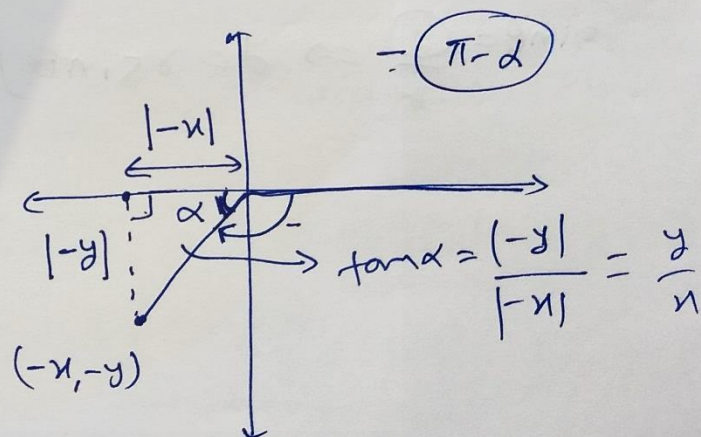
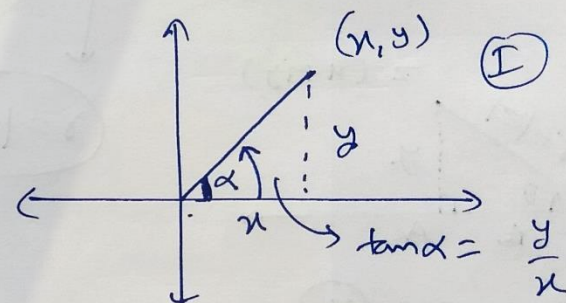
$$z = x + iy$$

(I) $\alpha = \tan^{-1} \left| \frac{y}{x} \right|$ ✓

(II)

Quadrant	$\theta = \text{P.A.}$
I	$\theta = \alpha$ ✓
II	$\theta = \pi - \alpha$ ✓
III	$\theta = \alpha - \pi$ ✓
IV	$\theta = -\alpha$ ✓

(III)



Cartesian Form

Polar Form

Extra

Euler's Form

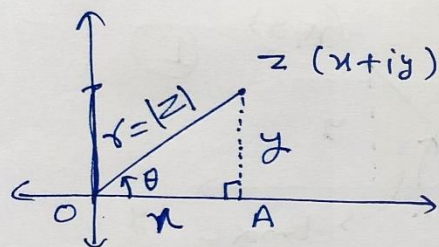
$$z = \boxed{x+iy} = r(\cos\theta + i\sin\theta) = r \cdot (e^{i\theta})$$

(x, y)

Cartesian
Coordinates

(r, θ)

Polar
Coordinates



$$\boxed{r = |z|}$$

$$\boxed{\theta = \arg(z)}$$

$$\left. \begin{aligned} \cos\theta &= \frac{\boxed{x}}{r} \Rightarrow x = r\cos\theta \\ \sin\theta &= \frac{\boxed{y}}{r} \Rightarrow y = r\sin\theta \end{aligned} \right\}$$

$$\left\{ \begin{aligned} e &= 2.718... \\ e^x &= \text{Exponential} \\ &\text{Fn.} \end{aligned} \right.$$

e.g. Convert $\frac{-1+i\sqrt{3}}{z}$ into Polar form.
 $r(\cos\theta + i\sin\theta)$

$$r = |z| = \sqrt{(-1)^2 + (\sqrt{3})^2}$$

$$= \sqrt{1+3} = 2 = r$$

$$\alpha = \tan^{-1}\sqrt{3}$$

$$\tan\alpha = \sqrt{3}$$

Argument (P.A).

$$\alpha = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right| = \tan^{-1}|\sqrt{3}| = \tan^{-1}\sqrt{3} = 60^\circ = \frac{\pi}{3}$$

Quadrant,

$$(-1+i\sqrt{3}) \equiv (-1, \sqrt{3}) \quad \text{II}$$

- +
+ +

$$r.e^{i\theta}$$

Argument $\theta = \pi - \alpha$
 $= \pi - \frac{\pi}{3}$

$$\theta = \frac{2\pi}{3}$$

Polar form: $r(\cos\theta + i\sin\theta) =$

$$\underline{-1+i\sqrt{3}} = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 2.e^{i\frac{2\pi}{3}}$$

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Class-11- Maths Exercise 5.2

Complex Number

Revision

$$Z = x + iy$$

Modulus

$$|Z| = r = \sqrt{x^2 + y^2}$$

Argument θ

$$(I) \alpha = \tan^{-1} \left| \frac{y}{x} \right|$$

(II)

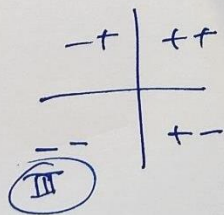
I	$\theta = \alpha$
II	$\theta = \pi - \alpha$
III	$\theta = \alpha - \pi$
IV	$\theta = -\alpha$

Q.1 $z = -1 - i\sqrt{3} \Rightarrow \begin{cases} x = -1 \\ y = -\sqrt{3} \end{cases}$

$$|z| = r = \sqrt{(-1)^2 + (-\sqrt{3})^2}$$

$$= \sqrt{1 + 3}$$

$$= 2$$



$$\arg(z) = \theta = ?$$

$$\alpha = \tan^{-1} \left| \frac{y}{x} \right| = \tan^{-1} \left| \frac{-\sqrt{3}}{-1} \right|$$

$$\alpha = \tan^{-1} |\sqrt{3}|$$

$$\tan \alpha = \sqrt{3}$$

Angle

$$?? \text{ (60°)}$$

$$60^\circ = \frac{\pi}{3}$$

$$z = -1 - i\sqrt{3} \rightarrow \text{III - quadrant.}$$

$$\arg(z) = \theta = \alpha - \pi$$

$$= \frac{\pi}{3} - \pi$$

$$= -\frac{2\pi}{3} \checkmark$$

$$|z| = 2$$

$$\arg(z) = -\frac{2\pi}{3}$$

Q.2. $z = -\sqrt{3} + i \rightarrow \begin{cases} x = -\sqrt{3} \\ y = 1 \end{cases}$

$$|z| = r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-\sqrt{3})^2 + 1^2}$$

$$= \sqrt{4}$$

$$\boxed{|z| = 2}$$

$$\alpha = \tan^{-1} \left| \frac{y}{x} \right|$$

$$\alpha = \tan^{-1} \left| \frac{1}{-\sqrt{3}} \right|$$

$$\alpha = \tan^{-1} \left| \frac{1}{\sqrt{3}} \right|$$

Angle $\rightarrow 30^\circ = \left(\frac{\pi}{6} \right)$

Quadrant. $(-\sqrt{3}, 1)$ $(-, +)$

$\textcircled{\text{II}}$

$$\arg(z) = \theta = \pi - \alpha$$

$$\theta = \pi - \frac{\pi}{6}$$

$$\boxed{\theta = \frac{5\pi}{6}}$$

modulus = 2 ✓

arg. = $\frac{5\pi}{6}$ ✓

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class -11 - maths. Exercise 5.2

Complex No.

Polar Form: $\underbrace{r(\cos\theta + i\sin\theta)}_{x+iy}$

$$r \rightarrow \text{modulus} = |z| = \sqrt{x^2 + y^2}$$

$$\theta \rightarrow \text{argument} = \begin{cases} \alpha = \tan^{-1}\left|\frac{y}{x}\right| \\ \text{Table (Quadrant)} \end{cases}$$

$$\left[\begin{array}{l} \text{I} \rightarrow \theta = \alpha \\ \text{II} \rightarrow \theta = \pi - \alpha \\ \text{III} \rightarrow \theta = \alpha - \pi \\ \text{IV} \rightarrow \theta = -\alpha \end{array} \right]$$

Q.3 $z = 1 - i$ $\begin{cases} x = 1 \\ y = -1 \end{cases} (x, y) \equiv (1, -1)$

$$r = |z| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

$$\alpha = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{-1}{1}\right| = \tan^{-1}1$$

Angle

$$\alpha = \tan^{-1}1$$

$$\alpha = \frac{\pi}{4} = 45^\circ$$

Quadrant: $(1, -1)$ $\left(\begin{smallmatrix} + \\ - \end{smallmatrix}\right)$

$\begin{array}{c} + \\ - \end{array}$ $\left(\begin{smallmatrix} + \\ - \end{smallmatrix}\right)$ IV

Argument $\theta = -\alpha$

$$\boxed{\theta = -\frac{\pi}{4}}$$

Polar Form: (r, θ)

$$\begin{aligned} & r(\cos\theta + i\sin\theta) \\ &= \sqrt{2} \left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right) \right) \end{aligned}$$

$$\textcircled{4} \quad z = -1 + i \quad \begin{matrix} \nearrow x = -1 \\ \searrow y = 1 \end{matrix} \quad (-1, 1)$$

Quadrant

II

$$\alpha = \tan^{-1} \left| \frac{y}{x} \right| = \tan^{-1} \left| \frac{1}{-1} \right|$$

$$\alpha = \tan^{-1} |1|$$

$$\alpha = \frac{\pi}{4} = 45^\circ$$

$$\begin{aligned} \text{Arg.} = \theta &= \pi - \alpha \\ &= \pi - \frac{\pi}{4} \end{aligned}$$

$$\theta = \frac{3\pi}{4} \quad \checkmark$$

$$|z| = r = \sqrt{x^2 + y^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$$\text{Polar form} = r(\cos \theta + i \sin \theta)$$

$$= \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$



$$\textcircled{5} \quad z = -1 - i \quad \begin{matrix} \nearrow x = -1 \\ \searrow y = -1 \end{matrix}$$

$$|z| = r = \sqrt{(-1)^2 + (-1)^2}$$

$$= \sqrt{2} \quad \checkmark$$

$\theta = \text{argument}$

$$\alpha = \tan^{-1} \left| \frac{y}{x} \right| = \tan^{-1} \left| \frac{-1}{-1} \right| = \tan^{-1} 1$$

$$\alpha = \frac{\pi}{4}$$

Quadrant: ~~II~~ $(x, y) = (-1, -1)$
III

$$\theta = \alpha - \pi$$

$$\theta = \frac{\pi}{4} - \pi = -\frac{3\pi}{4} \quad \checkmark$$

Polar form:

$$r(\cos \theta + i \sin \theta) = \sqrt{2} \left[\cos \left(-\frac{3\pi}{4} \right) + i \sin \left(-\frac{3\pi}{4} \right) \right]$$

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Ex 5.2

Complex Number

Q.6 $-3 \xrightarrow{\text{Convert}} \text{Polar Form } (r, \theta)$

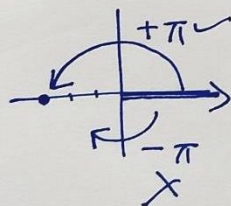
$$z = -3 = -3 + i.0 \quad \begin{cases} x = -3 \\ y = 0 \end{cases}$$

$$|z| = r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (0)^2} \\ = \sqrt{9} = 3$$

$$r = 3$$

Argument (θ) $(-\pi, \pi]$

Point $(-3, 0)$



$$\alpha = \tan^{-1} \left| \frac{y}{x} \right|$$

$$\theta = +\pi$$

$$r = 3$$

$$\theta = \pi$$

Polar Form:

$$r(\cos \theta + i \sin \theta) \\ = 3(\cos \pi + i \sin \pi) \checkmark$$

$$(7) z = \sqrt{3} + i = x + iy \quad (x, y) = (\sqrt{3}, 1)$$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = 2 \checkmark$$

Arg. $\theta = \alpha$

$$\alpha = \tan^{-1} \left| \frac{y}{x} \right|$$

(I)

$$\alpha = \tan^{-1} \left| \frac{1}{\sqrt{3}} \right| = \frac{\pi}{6}$$

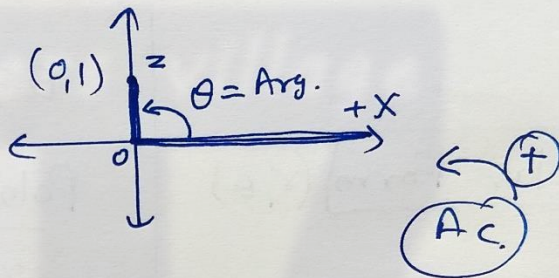
$$\theta = \frac{\pi}{6} \checkmark$$

Polar Form: $r(\cos \theta + i \sin \theta)$

$$= 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \checkmark$$

$$\textcircled{8} \quad z = i = \begin{matrix} 0 + i \cdot 1 \\ \uparrow \quad \nearrow \\ x + i \cdot y \end{matrix} \quad \underline{x=0, y=1}$$

Point $(x, y) \equiv (0, 1)$



Argument $= \theta = +\frac{\pi}{2} = 90^\circ$

$$|z| = r = \sqrt{x^2 + y^2} = \sqrt{0^2 + 1^2} = \sqrt{1} = 1$$

Polar Form: $r(\cos \theta + i \sin \theta)$

$$= 1 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$= \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \quad \checkmark$$

Complex Numbers & Quadratic Equations.

Exercise 5.3

$$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{D}}{2a} \quad \sqrt{D} \geq 0 \quad \begin{matrix} \sqrt{+} \\ \sqrt{-} \end{matrix}$$

Roots.

$$D = \text{Discriminant} = b^2 - 4ac$$

$$\boxed{D \geq 0} \checkmark$$

Theory: Fundamental Theorem of Algebra $\rightarrow$

"A polynomial Equation has at least one root"

"A polynomial equation of degree n has n roots"

Note:

① A quadratic equation has maximum 2 real roots.

② A quadratic equation has exactly 2 roots.

Solve Ex. 5.3

$x = ?$ roots = ?

① $x^2 + 3 = 0$

$$\Rightarrow x^2 = -3$$

$$\Rightarrow x = \pm \sqrt{-3}$$

$$x = \pm i\sqrt{3}$$

$$\left. \begin{array}{l} x_1 = i\sqrt{3} \\ x_2 = -i\sqrt{3} \end{array} \right\} \text{roots}$$

$\sqrt{-1} = i$

② $2x^2 + x + 1 = 0$

$a = 2, b = 1, c = 1$

Quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1 - 8}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{-7}}{4} = \frac{-1 \pm \sqrt{7}i}{4}$$

$$\boxed{3} \quad x^2 + 3x + 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{9 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$x = \frac{-3 \pm \sqrt{-27}}{2}$$

$$27 = 3 \times 3 \times 3$$

$$\boxed{x = \frac{-3 \pm i \cdot 3 \cdot \sqrt{3}}{2}}$$

$$\boxed{4} \quad -x^2 + x - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-1)(-2)}}{2(-1)}$$

$$x = \frac{-1 \pm \sqrt{1 - 8}}{-2}$$

$$x = \frac{-1 \pm \sqrt{-7}}{-2}$$

$$\boxed{x = \frac{-1 \pm i\sqrt{7}}{-2}}$$

Toppers Village:

Exercise 5.3 * Complex Number

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Q.5 $x^2 + 3x + 5 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$

$$x = \frac{-3 \pm \sqrt{9 - 20}}{2}$$

$$x = \frac{-3 \pm \sqrt{-11}}{2}$$

$$x = \frac{-3 \pm i\sqrt{11}}{2}$$

Q.6 $x^2 - x + 2 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 8}}{2}$$

$$x = \frac{1 \pm \sqrt{-7}}{2}$$

$$x = \frac{1 \pm i\sqrt{7}}{2}$$

Q.7 $\sqrt{2}x^2 + x + \sqrt{2} = 0$

$a = \sqrt{2}$ $b = 1$ $c = \sqrt{2}$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot \sqrt{2} \cdot \sqrt{2}}}{2 \cdot \sqrt{2}}$$

$$x = \frac{-1 \pm \sqrt{1 - 8}}{2 \cdot \sqrt{2}}$$

$$x = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}}$$

$$x = \frac{-1 \pm i\sqrt{7}}{2\sqrt{2}}$$

Q.8

$$\sqrt{3} x^2 - \sqrt{2} x + 3\sqrt{3} = 0$$

$$x = \frac{-(-\sqrt{2}) \pm \sqrt{(-\sqrt{2})^2 - 4 \cdot \sqrt{3} \cdot 3\sqrt{3}}}{2 \cdot \sqrt{3}}$$

$$x = \frac{\sqrt{2} \pm \sqrt{2 - 36}}{2\sqrt{3}}$$

$$x = \frac{\sqrt{2} \pm \sqrt{-34}}{2\sqrt{3}}$$

$$x = \frac{\sqrt{2} \pm i\sqrt{34}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 - 2\sqrt{2}}}{2}$$

$$4 = 2 \times 2 \\ = 2 \times \sqrt{2} \cdot \sqrt{2}$$

Q.9

$$x^2 + x + \frac{1}{\sqrt{2}} = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot \frac{1}{\sqrt{2}}}}{2 \cdot 1}$$

$$x = \frac{-1 \pm \sqrt{1 - \frac{4}{\sqrt{2}}}}{2}$$

$$x = \frac{-1 \pm \sqrt{\frac{\sqrt{2} - 4}{\sqrt{2}}}}{2}$$

$$\Rightarrow x = \frac{-1 \pm i \sqrt{\frac{4 - \sqrt{2}}{\sqrt{2}}}}{2}$$

$$\Rightarrow x = \frac{-1 \pm i \sqrt{4 - \sqrt{2}}}{2\sqrt{2}}$$

Q.10

$$x^2 + \frac{x}{\sqrt{2}} + 1 = 0$$

$$x = \frac{-\frac{1}{\sqrt{2}} \pm \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$x = \frac{-\frac{1}{\sqrt{2}} \pm \sqrt{\frac{1}{2} - 4}}{2}$$

$$x = \frac{-\frac{1}{\sqrt{2}} \pm \sqrt{\frac{-7}{2}}}{2}$$

$$x = \left(\frac{-1 \pm i\sqrt{7}}{\sqrt{2}} \right) \cdot \frac{1}{2}$$

$$x = \frac{-1 \pm i\sqrt{7}}{2\sqrt{2}}$$

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Class-11-Maths Miscellaneous Exercise [5]

Complex Numbers

[Q.1] Evaluate $\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$i^{4n} = 1$	$i^{4n+2} = -1$
$i^{4n+1} = i$	$i^{4n+3} = -i$

$$i^{18} = i^{16+2} = i^{4 \cdot 4 + 2} = -1$$

$$\left(\frac{1}{i} \right)^{25} = \frac{1}{i^{25}} = \frac{1}{i^{24+1}} = \frac{1}{i} \times \frac{i}{i}$$

$$= \frac{i}{i^2} = \frac{i}{-1} = -i$$

$$\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3 = (-1 - i)^3 = -(1+i)^3$$

$(a+b)^3$

$$(a+b)^3 = [a^3 + b^3 + 3a^2b + 3ab^2]$$
$$-(1+i)^3 = -[1 + i^3 + 3 \cdot 1^2 \cdot i + 3 \cdot 1 \cdot i^2]$$

$$= -[1 - i + 3i - 3]$$

$$= -[-2 + 2i]$$

$$= 2 - 2i$$

[Q.2] Complex Numbers z_1, z_2

To Prove:

$$\text{Re}(z_1 z_2) = \underbrace{\text{Re } z_1}_{x_1} \cdot \underbrace{\text{Re } z_2}_{x_2} - \underbrace{\text{Im } z_1}_{y_1} \cdot \underbrace{\text{Im } z_2}_{y_2}$$

Let $\boxed{z_1 = x_1 + iy_1}$

$\boxed{z_2 = x_2 + iy_2}$

$$\text{RHS} = x_1 \cdot x_2 - y_1 \cdot y_2$$

$$\text{LHS} = \text{Re}(z_1 z_2)$$

$$= \text{Re}[(x_1 + iy_1) \cdot (x_2 + iy_2)]$$

$$\text{LHS} = \text{Re}(x_1(x_2 + iy_2) + iy_1(x_2 + iy_2))$$

$$= \text{Re} \left[\begin{array}{cccc} x_1 x_2 & + & x_1 \cdot i \cdot y_2 & + & i y_1 x_2 & + & i^2 \cdot y_1 y_2 \end{array} \right]$$

$$= \text{Re} \left[\underbrace{x_1 x_2 - y_1 y_2} + i \underbrace{(x_1 y_2 + y_1 x_2)} \right]$$

$$= x_1 x_2 - y_1 y_2$$

$$= \text{Re} z_1 \cdot \text{Re} z_2 - \text{Im} z_1 \cdot \text{Im} z_2$$

$$= \text{RHS.}$$

③ Reduce $\left(\frac{1}{1-4i} \right) - \left(\frac{2}{1+i} \right) \cdot \left[\frac{3-4i}{5+i} \right]$ into standard form.

$$\frac{1}{1-4i} \times \frac{1+4i}{1+4i} = \frac{1+4i}{1^2 - (4i)^2} = \frac{1+4i}{1+16} = \frac{1+4i}{17}$$

$$\frac{2}{1+i} \times \frac{1-i}{1-i} = \frac{2-2i}{1-i^2} = \frac{2-2i}{1+1} = 1-i$$

$$\frac{3-4i}{5+i} \times \frac{5-i}{5-i} = \frac{15-3i-20i-4}{5^2 - i^2}$$

$$= \frac{11-23i}{26}$$

$$\left(\frac{1}{1-4i} - \left(\frac{2}{1+i} \right) \right) \cdot \left(\frac{3-4i}{5+i} \right)$$

$$= \left(\frac{1+4i}{17} - 1+i \right) \cdot \left(\frac{11-23i}{26} \right)$$

$$= \left(\frac{1+4i-17+17i}{17} \right) \cdot \left(\frac{11-23i}{26} \right)$$

$$= \frac{-16+21i}{17} \times \frac{11-23i}{26}$$

$$= \frac{-176 + \overbrace{408i}^{368i} + 231i + 483}{442}$$

$$= \frac{307 + \overbrace{599i}^{399i}}{442}$$

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Class - 11 - Maths Miscellaneous Exercise [5]

Complex No.

④ $x-iy = \sqrt{\frac{a-ib}{c-id}}$ (Given)

To Prove $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Concept $|x+iy| = \sqrt{x^2+y^2}$

Given: $x-iy = \sqrt{\frac{a-ib}{c-id}}$ $|z|^2 = |z^2|$

Square: $(x-iy)^2 = \left(\frac{a-ib}{c-id}\right)$

Modulus $| (x-iy)^2 | = \left| \frac{a-ib}{c-id} \right|$

Properties

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$$

$$\Rightarrow |(x-iy) \cdot (x-iy)| = \left| \frac{a-ib}{c-id} \right|$$

$$\Rightarrow |x-iy| \cdot |x-iy| = \frac{|a-ib|}{|c-id|}$$

$$\Rightarrow \sqrt{x^2+y^2} \cdot \sqrt{x^2+y^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$$

$\Rightarrow$ Square $\sqrt{c^2+d^2}$

$$(x^2+y^2)^2 = \left(\frac{a^2+b^2}{c^2+d^2} \right)$$

⑤ Polar Form: $r(\cos\theta + i\sin\theta)$

(i) $\frac{1+7i}{(2-i)^2} \rightarrow$ Simplified Form $\left| \begin{array}{l} r=|z| \\ \theta = \arg. \end{array} \right.$

$(a+ib)$ $i^2 = -1$

$$= \frac{1+7i}{2^2 + i^2 - 4i} = \frac{1+7i}{3-4i}$$

$$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i-28}{3^2 - (4i)^2}$$

$$= \frac{-25+25i}{9+16} = \frac{-25+25i}{25}$$

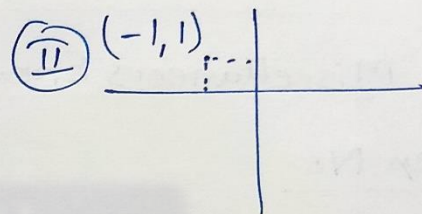
$$z = \boxed{-1+i} = x+iy \quad (x,y) \equiv (-1,1)$$

$$r=|z| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

$$\theta = \arg(z) = \text{Principal Argument.}$$

$$\rightarrow \alpha = \tan^{-1} \left| \frac{y}{x} \right| = \tan^{-1} \left| \frac{1}{-1} \right| = \tan^{-1}(1)$$

$$\alpha = 45^\circ = \frac{\pi}{4} \checkmark$$



$$\arg(z) = \theta = \pi - \alpha$$

$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \checkmark$$

Polar Form =

$$= r(\cos\theta + i\sin\theta)$$

$$= \sqrt{2} \left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} \right)$$

Toppers Village Miscellaneous Exercise 15 Complex No.

Q.6 $(3x^2 - 4x + \frac{20}{3} = 0) \times 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$9x^2 - 12x + 20 = 0$

$$x = \frac{-(-12) \pm \sqrt{144 - 4 \cdot 9 \cdot 20}}{2 \times 9}$$

$$x = \frac{12 \pm \sqrt{144 - 720}}{18}$$

$$x = \frac{12 \pm \sqrt{-576}}{18}$$

$$x = \frac{12 \pm i \cdot 24}{18}$$

$$x = \frac{2}{3} \pm i \frac{4}{3}$$

Q.7 $x^2 - 2x + \frac{8}{2} = 0$

$$x = \frac{-(-2) \pm \sqrt{4 - 4 \cdot 1 \cdot \frac{8}{2}}}{2 \times 1}$$

$$x = \frac{2 \pm \sqrt{4 - 6}}{2}$$

$$x = \frac{2 \pm \sqrt{-2}}{2}$$

$$x = \frac{2 \pm i\sqrt{2}}{2}$$

$$x = 1 \pm \frac{i}{\sqrt{2}}$$

Q.8 $27x^2 - 10x + 1 = 0$

$$x = \frac{-(-10) \pm \sqrt{100 - 4 \cdot 27 \cdot 1}}{2(27)}$$

$$x = \frac{10 \pm \sqrt{100 - 108}}{54}$$

$$x = \frac{10 \pm \sqrt{-8}}{54}$$

$$x = \frac{10 \pm i2\sqrt{2}}{54}$$

$$x = \frac{5 \pm i\sqrt{2}}{27}$$

Q.9 $21x^2 - 28x + 10 = 0$

$$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4 \cdot 21 \cdot 10}}{2(21)}$$

$$x = \frac{28 \pm \sqrt{784 - 840}}{42}$$

$$x = \frac{28 \pm \sqrt{-56}}{42}$$

$$x = \frac{28 \pm i\sqrt{4 \times 14}}{42}$$

$$x = \frac{28 \pm 2\sqrt{14} \cdot i}{42}$$

$$x = \frac{14 \pm \sqrt{14} \cdot i}{21}$$

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Miscellaneous Exercise [5]. Complex No.

⑩ If $z_1 = 2 - i$, $z_2 = 1 + i$,
Find $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right|$. $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$ ★

$$\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right| = \left| \frac{2 - \cancel{i} + 1 + \cancel{i} + 1}{2 - i - 1 - \cancel{i} + \cancel{i}} \right|$$

$$= \frac{|4|}{|1 - i|}$$

$$\begin{array}{cc} \sqrt{4^2 + 0^2}, & |4| \\ \downarrow & \downarrow \\ 4 & 4 \end{array}$$

$$= \frac{4}{\sqrt{1^2 + (-1)^2}} = \frac{4}{\sqrt{2}} = \frac{(2 \times \cancel{\sqrt{2}} \cdot \cancel{\sqrt{2}})}{\cancel{\sqrt{2}}}$$

$$= 2\sqrt{2}$$

⑪ $a + ib = \frac{(x + i)^2}{2x^2 + 1}$ Given

To Prove: $a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$

$$a + ib = \frac{(x + i)^2}{2x^2 + 1}$$

By taking modulus Both Sides.

$$|a + ib| = \left| \frac{(x + i)^2}{2x^2 + 1} \right|$$

$(z_1 \cdot z_2) = |z_1| \cdot |z_2|$

$$\Rightarrow \sqrt{a^2 + b^2} = \frac{|x + i|^2}{|2x^2 + 1 + 0 \cdot i|}$$

$$\Rightarrow \sqrt{a^2 + b^2} = \frac{(\sqrt{x^2 + 1^2})^2}{\sqrt{(2x^2 + 1)^2 + 0^2}}$$

$$\Rightarrow \sqrt{a^2+b^2} = \frac{\left(\sqrt{x^2+1^2}\right)^2}{\sqrt{(2x^2+1)^2+0^2}}$$

By squaring both sides.

$$\Rightarrow \left(\sqrt{a^2+b^2}\right)^2 = \left[\frac{x^2+1}{2x^2+1}\right]^2$$

$$\Rightarrow a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$$

[12] $z_1 = 2-i$ $z_2 = -2+i$

(i) $\operatorname{Re}\left(\frac{z_1 z_2}{z_1}\right)$

$$= \operatorname{Re}\left(\frac{(2-i) \cdot (-2+i)}{2-i}\right)$$

$$= \operatorname{Re}\left(\frac{-4+2i+2i-\overset{-1}{\textcircled{i^2}}}{2+i}\right) \quad \textcircled{i^2 = -1}$$

$$= \operatorname{Re}\left(\frac{-4+1+4i}{2+i} \times \frac{2-i}{2-i}\right)$$

$$= \operatorname{Re}\left(\frac{(-3+4i) \cdot (2-i)}{2^2-(i)^2}\right)$$

$$= \operatorname{Re}\left(\frac{-6+3i+8i-4\overset{-1}{\textcircled{i^2}}}{4+1}\right)$$

$$= \operatorname{Re}\left(\frac{-2+11i}{5}\right)$$

$$= \underline{\underline{\operatorname{Re}\left(\textcircled{-\frac{2}{5}} + \textcircled{\frac{11i}{5}}\right)}}$$

$$= -\frac{2}{5} \quad \checkmark$$

$$\begin{aligned}
 (12) \quad (ii) \quad \operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right) \\
 &= \operatorname{Im} \left(\frac{1}{(2-i) \cdot (2-i)} \right) \\
 &= \operatorname{Im} \left(\frac{1}{(2-i) \cdot (2+i)} \right) \\
 &= \operatorname{Im} \left(\frac{1}{2^2 - (i)^2} \right) \\
 &= \operatorname{Im} \left(\frac{1}{4 - (-1)} \right) \\
 &= \operatorname{Im} \left(\frac{1}{5} \right) \\
 &= \operatorname{Im} \left(\frac{1}{5} + 0i \right) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 (13) \quad z &= \frac{1+2i}{1-3i} \\
 \text{Modulus} &= |z| = \left| \frac{1+2i}{1-3i} \right| = \frac{|1+2i|}{|1-3i|} \\
 &= \frac{\sqrt{1^2+2^2}}{\sqrt{1^2+(-3)^2}} \\
 &= \frac{\sqrt{5}}{\sqrt{10}} = \frac{\sqrt{5}}{\sqrt{2 \cdot 5}} = \frac{1}{\sqrt{2}} \checkmark \\
 \text{Argument (P.A.)} \\
 z &= \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2-(3i)^2} \\
 &= \frac{-5+5i}{1+9} = \frac{-5+5i}{10} = -\frac{5}{10} + \frac{5i}{10} \\
 &= -\frac{1}{2} + \frac{i}{2}
 \end{aligned}$$

$$z = \frac{1+2i}{1-3i} = -\frac{1}{2} + \frac{i}{2} = x + iy$$

argument of $z = \theta = ?$

$$\alpha = \tan^{-1} \left| \frac{y}{x} \right| = \tan^{-1} \left| \frac{\frac{1}{2}}{-\frac{1}{2}} \right|$$

$$\checkmark \alpha = \tan^{-1} |-1| = \tan^{-1}(1) = 45^\circ = \left(\frac{\pi}{4} \right)$$

$$(x, y) \equiv \left(-\frac{1}{2}, \frac{1}{2} \right) \quad \begin{matrix} \text{II} \\ (-, +) \end{matrix}$$

II- quadrant

$$\arg(z) = \pi - \alpha$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4} \checkmark$$

$$r = |z| = \frac{1}{\sqrt{2}}$$

$$\arg(z) = \frac{3\pi}{4} = \theta$$

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Class - 11 - Complex No. Misc. Ex. 5

[14] $(x-iy)(3+5i) = \text{conjugate of } \underline{-6-24i}$

$$\Rightarrow (x-iy)(3+5i) = \overline{(-6-24i)}$$

$$\Rightarrow 3x + \underbrace{5x.i - 3y.i}_{-1} - 5.y.\underbrace{(i^2)}_{-1} = -6 + 24i$$

$$\Rightarrow \underbrace{(3x+5y)} + i \underbrace{(5x-3y)} = -6 + 24i$$

By Comparision

Real Part

$$3x + 5y = -6 \quad \text{--- (1) } \times 3 \Rightarrow 9x + 15y = -18$$

Im. Part

$$5x - 3y = 24 \quad \text{--- (2) } \times 5 \Rightarrow \begin{array}{r} 25x - 15y = 120 \\ + \\ 9x + 15y = -18 \\ \hline 34x = 102 \end{array}$$

$$x = \frac{102}{34} = 3$$

$$\underline{x=3}$$

eqn (1) : ->

$$3x + 5y = -6$$

$$\Rightarrow 9 + 5y = -6$$

$$\Rightarrow 5y = -15$$

$$\boxed{y = -3}$$

$x = 3$
$y = -3$

15 modulus of $\left(\frac{1+i}{1-i} - \frac{1-i}{1+i}\right)$

$$= \left| \frac{1+i}{1-i} - \frac{1-i}{1+i} \right|$$

$$\cancel{|z_1 - z_2| = |z_1| - |z_2|}$$

$$= \left| \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)} \right|$$

$$= \left| \frac{(1+i^2 + 2 \cdot 1 \cdot i) - (1+i^2 - 2 \cdot 1 \cdot i)}{(1-i)(1+i)} \right|$$

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$$

$$|z_1 z_2| = |z_1| \cdot |z_2|$$

$$= \frac{|\cancel{1} + \cancel{i^2} + 2i - \cancel{1} - \cancel{i^2} + 2i|}{|1-i| \cdot |1+i|}$$

$$4i = 0 + 4i'$$

$$= \frac{|4i|}{\sqrt{1^2 + (-1)^2} \cdot \sqrt{1^2 + 1^2}} = \frac{\sqrt{0^2 + 4^2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{4}{2} = 2 \checkmark$$

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$$(x+iy)^3 = u+iv$$

To Prove

$$\boxed{\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)}$$

$$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$\Rightarrow x^3 + (iy)^3 + 3 \cdot x^2 \cdot iy + 3 \cdot x \cdot (iy)^2 = u+iv$$

$$\underbrace{i^2 = -1}, \quad \underbrace{i^3 = -i}$$

$$\Rightarrow \underbrace{x^3}_{\uparrow} + \underbrace{(-i)y^3}_{\uparrow} + \underbrace{3x^2 \cdot y \cdot i}_{\uparrow} + \underbrace{3xy^2(-1)}_{\uparrow} = u+iv$$

$$\Rightarrow \left(\underbrace{x^3 - 3xy^2}_{\uparrow} \right) + i \left(\underbrace{3x^2y - y^3}_{\uparrow} \right) = \underbrace{u+iv}_{\uparrow}$$

By Comparison

$$u = x^3 - 3xy^2$$

$$v = 3x^2y - y^3$$

$$LHS = \frac{u}{x} + \frac{v}{y}$$

$$= \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$$

$$= \underline{x^2} - \underline{3y^2} + \underline{3x^2} - \underline{y^2}$$

$$= 4x^2 - 4y^2$$

$$= 4(x^2 - y^2)$$

$$= RHS.$$

H.P.

Class - 11 - Complex No. Misc. Ex. [5]

[Q.17] ★

$\alpha, \beta \rightarrow$ Complex No.

$|\beta| = 1$

$\left| \frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right| = ?$

Theory.

★ $\boxed{z \cdot \bar{z} = |z|^2} \Leftrightarrow$

Results.

$\beta \cdot \bar{\beta} = |\beta|^2$

$\boxed{\beta \cdot \bar{\beta} = 1}$

Properties

$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$ ✓

$\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$ ✓

$\overline{\bar{\alpha}} = \alpha$

$\alpha = x + iy$
 $\bar{\alpha} = x - iy$
 $\overline{\bar{\alpha}} = x + iy = \alpha$

$|z|^2 = z \cdot \bar{z}$ ✓

put $z = \frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta}$

$\Rightarrow \left| \frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right|^2 = \left(\frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right) \cdot \left(\frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right)$

$= \left(\frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right) \cdot \left(\frac{\bar{\beta} - \bar{\alpha}}{1 - \alpha \cdot \bar{\beta}} \right)$



$$\Rightarrow \left| \frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right|^2 = \left(\frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right) \cdot \left(\frac{\bar{\beta} - \bar{\alpha}}{1 - \alpha \cdot \bar{\beta}} \right)$$

$$= \frac{\beta \cdot \bar{\beta} - \beta \cdot \bar{\alpha} - \alpha \cdot \bar{\beta} + \alpha \cdot \bar{\alpha}}{1 - \alpha \cdot \bar{\beta} - \bar{\alpha} \cdot \beta + \alpha \cdot \bar{\alpha} \cdot (\beta \cdot \bar{\beta})}$$

Put $\beta \cdot \bar{\beta} = 1$

$$= \frac{(1 - \beta \cdot \bar{\alpha} - \alpha \cdot \bar{\beta} + \alpha \cdot \bar{\alpha})}{(1 - \alpha \cdot \bar{\beta} - \bar{\alpha} \cdot \beta + \alpha \cdot \bar{\alpha})}$$

$$\Rightarrow \left| \frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right|^2 = 1$$

$$\Rightarrow \left| \frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right| = \pm 1 = +1$$

$$\begin{aligned} n^2 &= 1 \\ n &= \pm 1 \end{aligned}$$

$$|z| = 1 \quad \text{X}$$

$$\boxed{\left| \frac{\beta - \alpha}{1 - \bar{\alpha} \cdot \beta} \right| = 1}$$

Q.18 Find the no. of non zero integral solutions of the equation $|1-i|^x = 2^x$.

Integers
 $\dots -3, -2, -1, 0, 1, 2, 3 \dots$
 $\uparrow \qquad \qquad \qquad \uparrow$

$$|1-i|^x = 2^x$$

$$\Rightarrow (\sqrt{1^2 + (-1)^2})^x = 2^x$$

$$\Rightarrow (\sqrt{2})^x = 2^x$$

$$\sqrt{2} = (2)^{\frac{1}{2}}$$

$$(a^m)^n = a^{m \cdot n}$$

$$\Rightarrow \left(2^{\frac{1}{2}}\right)^x = 2^x$$

$$\Rightarrow 2^{\frac{x}{2}} = 2^x$$

$$\Rightarrow \frac{x}{2} = x$$

$$\Rightarrow x = 2x$$

$$\Rightarrow \boxed{0 = x} \quad \checkmark$$

only

Non-zero Solⁿ = No Solⁿ,

No. of non-zero solutions = 0

①9 To Prove, $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

Given, $(a+ib)(c+id)(e+if)(g+ih) = A+iB$

By taking modulus Both sides:

$$(z_1 \cdot z_2) = |z_1| |z_2|$$

$$\Rightarrow |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$$

$$\Rightarrow |a+ib| \cdot |c+id| \cdot |e+if| \cdot |g+ih| = |A+iB|$$

$$\Rightarrow \sqrt{a^2+b^2} \cdot \sqrt{c^2+d^2} \cdot \sqrt{e^2+f^2} \cdot \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$$

Square

$$\Rightarrow (a^2+b^2) \cdot (c^2+d^2) \cdot (e^2+f^2) \cdot (g^2+h^2) = (A^2+B^2)$$

20 If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least integral ^{Natural} value of m .

$\{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \dots\}$

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left[\frac{(1+i) \times (1+i)}{(1-i) \times (1+i)}\right]^m = 1$$

$$\Rightarrow \left[\frac{(1+i)^2}{1^2 - i^2}\right]^m = 1$$

$$\Rightarrow \left[\frac{1^2 + i^2 + 2 \cdot 1 \cdot i}{1 + 1}\right]^m = 1$$

$$\Rightarrow \left(\frac{\cancel{1} - \cancel{1} + 2i}{2}\right)^m = 1$$

$$\Rightarrow \boxed{(i)^m = 1}$$

$$i^{-12} = 1 \quad \checkmark$$

$$i^{-8} = 1 \quad \times$$

$$i^{-4} = 1 \quad \times$$

$$i^0 = 1 \quad \times$$

$$i^4 = 1$$

$$i^8 = 1$$

$$i^{12} = 1$$

$\vdots$

$$i^{-4} = \frac{1}{i^4} = \frac{1}{1} = 1$$

Ans. = 4

Natural

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>